



# Individual Innovative Teaching and Learning Awards 2018

Full Application

**Applicant Name(s):**

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**Course(s):**

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**Project Title:**

The Art of Math

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**Project Goals:**

- Further develop a new mathematics course, successfully piloted in Spring 2017, to enhance its active learning elements and to prepare scaling the course for larger enrolments in future iterations.
- Expand the course's cultural diversity to include new sources like Gerardo Aldana's work on Mesoamerican astronomy and Ron Eglash's African Fractals.
- Improve quality and number of small-group in-class activities (additional origami activities, use of historical teaching instruments like the monochord and the abacus, and other teaching tools that can be brought into the classroom).
- Plan and implement hands-on individual and small-group activities to be done outside of class time (new tools in Canvas, enhanced interactive tools like Geogebra notebooks, guided student-developed software tools, 3D printing projects, architecture study using structures on campus).
- Create new interactive online pre-lecture Canvas modules designed to introduce students to the concepts and skills they will build upon during in-class activities.

## Overview of innovation idea:

This math course, which the Department of Mathematics has approved to fulfill the A&S MQR requirement, pursues a new way to teach mathematical reasoning as a human endeavor of creative, intellectual exploration, whose cultural variations are as important as the cultural commonalities. We teach Greek geometry alongside Chinese algorithmic geometry, Greek astronomy along with astronomical texts from the Sanskrit and medieval Islamic traditions, and number theory and harmonics from the medieval Islamic and classical Chinese traditions along with authors from medieval Europe.

To our knowledge, this MQR course is unique within the humanities at Cornell, and it has already piqued considerable interest both at Cornell and at comparable institutions (e.g., we've had inquiries about our syllabus from UC - Berkeley, McGill University, University of Michigan, and Newcastle University, UK).

The course is structured as two 75-minute meetings per week, with each course meeting divided between interactive lecture and hands-on activities. The first time we offered the course, we used tools like Blackboard group discussions and notebooks in Geogebra (a freeware tool for creating step-by-step solutions to mathematics problems, animations, etc.) to move content, additional student engagement, and in-depth instructor feedback outside the meeting time in order to make more room for hands-on activities in small groups. These activities included geometry problem-solving with origami, collaborative solutions to problems given in class, hands-on work with monochords (a single-stringed musical instrument used to demonstrate harmonic intervals), and even games like Rithmomachia, a medieval number-theory game (Cornell Chronicle: Class teaches math and music with hands-on approach.) These activities were enjoyable, but also rigorous and challenging, an essential component of the course that we would like to develop further.

At the same time, we would like to complement the expansion of in-class activities with an expanded set of interactive activities to be done outside of class, including developing interactive Geogebra notebooks (or equivalent tools using different software, if applicable) for student use, devising projects using Cornell resources like the 3D printing labs, and so on. During our initial CTI consultation, we also discussed working with an assistant to develop additional Geogebra notebooks that will include video explanations from the instructors, along with the mathematical content for pre-lecture preparation. All of these will allow more classroom time to be spent on group activities. Developing these activities and strengthening the course's existing active-learning elements – built around sustainable, scalable, and repeatable activities – is the core of what the grant will enable us to achieve.

## Objectives:

We target all five CTI innovation objectives:

- CTI: Develop innovative pedagogical strategies for the undergraduate learning environment that actively engage students and improve their success.
  - The Art of Math: *develop new, flexible strategies for teaching mathematics that will encourage students to engage in collaborative, creative work while we maintain a rigorous, detailed approach to teaching proof structures and other problem-solving strategies.*
- Explore novel ways of creatively/critically thinking about disciplines, examining diverse perspectives and processes from across disciplines, and integrating varying approaches to solving complex problems for Cornell undergraduates.
  - The Art of Math: *integrate mathematical practices and pedagogy with humanities and the arts, encompassing music, architecture, design, and the visual arts as they differently engage the principles of mathematical harmonics, geometry, astronomy, and number theory*
- Reimagine the undergraduate learning experience within a department or improve teaching and learning at a university-wide level.
  - The Art of Math: *reimagines the (current) MQR requirement for A&S and offers students in all Colleges an opportunity to reimagine mathematical reasoning from a perspective that integrates the humanities and STEM*
- Employ inclusive learning practices.
  - The Art of Math: *presents mathematics as an inclusive human endeavor, stressing variations in mathematical practices across cultures by using Greek, Latin, Sanskrit, Chinese, Arabic, and Persian sources*
- Build upon research-based learning and teaching practices
  - *integrates active learning components into classroom work following methods tested and documented by Cornell's Active Learning Initiative*

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## Learning Outcomes:

- formulate proofs of geometrical problems (both pure and applied) from different traditions (e.g. Greek, Chinese, Sanskrit)
- calculate harmonic ratios from different traditions (e.g. Greek, Latin, Chinese, Persian)
- develop visual reasoning skills for mathematical problem-solving

- collaborate in developing and assessing problem-solving strategies
  - connect shared patterns of mathematical reasoning across diverse mathematical domains
  - compare techniques of mathematical reasoning from Western traditions with those from the Islamic world, China, and India
  - design tools (diagrams, computer programs, concrete objects, etc.) to explain concepts of symmetry and proportion to peers and instructors
  - recognize principles of symmetry, harmony, and proportion as applied in environments both local (e.g. architecture on Cornell's campus) and global (e.g. non-Western musical traditions)
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## Perspective on Innovation:

The assessments for the course include contributions to the small-group journals (emphasizing thoughtful engagement with problem-solving strategies rather than simply arriving at the “right” answer), weekly homework problem sets (submitted and graded individually before being posted to the group), contributions to discussions in lecture and classroom activities, and a final project carried out by the members of each Blackboard journal group. Students were required to discursively assess their peers for the final group project, which produced useful self-reflections as well as assessments of the peer projects. We were satisfied with the approach to the study of mathematics these assessments encouraged, as students came to appreciate mathematics as a rich set of culturally variant activities rather than an all-or-nothing search for a single correct approach.

Through the support of the innovation grant, we would be able to work with a graduate student or postdoctoral assistant to develop and integrate more assessments involving design and construction alongside the verbal and diagrammatic problem sets we offered in the first iteration of the course: these could include, for example, software tools designed to explore their work in astronomy and harmonics or 3D-printed representations of the geometrical and architectural structures studied in class. These additional avenues for assessment would reinforce our goal of making the course welcoming to a wide range of students by valuing students’ design and construction skills throughout the course, rather than just in selected classroom activities and final project plans.

## Assessment and Evaluation:

We will assess the impact of incorporating the new materials into the course through a combination of quantitative and qualitative assessments including:

- 1) quantitative scores from the course rubrics for individual problem sets and discussion-journal contributions (see below for examples),
- 2) evaluation of student reflections and discussions in journals,
- 3) student participation and performance in in-class activities, and
- 4) quantitative scores and qualitative assessments of final group projects.

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## Impact on teaching and Learning:

The course contributes to new developments in STEM teaching at Cornell in several important ways. By learning to think about mathematics as a creative, collaborative problem-solving process, students who might otherwise experience mathematics as alienating or intimidating instead came to think about it as informing familiar experiences like listening to music or walking into a building designed in a classical style, an approach we reinforced with in-class games and activities like origami. As one student observed in her/his evaluation, "Really enjoyed this class and especially appreciated that it was a space to explore concepts and new areas without fear of judgement."

Weekly problem sets encouraged students to combine rigor and creativity in designing their own mathematically informed structures, for example designing their own tuning systems, creating a series of diagrams to explain the most important features of Archimedes' Method, or devising models and explanations for Kepler's model of the cosmos to convince a patron to continue funding his work. Combinations of rigorous full proofs and "plain English" explanations helped students to connect the intricacies of mathematical reasoning to the broader concepts behind the problems. In considering the structures of argumentative and proof used in different mathematical cultures rather than just looking at their results, students were encouraged to think about mathematical argumentation as akin to other forms of evidence-based argumentative discourse, rather than a special form of reasoning totally separate from other domains of intellectual and practical life.

The overall effect of these approaches was to create a welcoming environment for a diverse group of students. We had success attracting and retaining students from across the humanities, arts, and STEM disciplines, with a good gender balance and a wide range of national and ethnic backgrounds. It

might be the case that other STEM courses where diversity remains a desideratum could benefit from adapting some of our pedagogical methods into their coursework and assessment schemes.

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## Scale:

First offered in Spring 2017, the course brought together a diverse group of students from humanities (e.g. Classics, History, Music), STEM (e.g. Electrical Engineering, Mechanical Engineering, Mathematics), and the arts (Music, Architecture), representing academic levels from freshmen to seniors. In future iterations (it is scheduled to be offered again in Spring 2019), we hope to maintain this diverse makeup, while expanding enrollment using new materials we hope to develop with the help of the innovation grant.

While the enrollment for the first pilot offering was capped at 25, we designed central components of the course to be scalable to larger enrollments. For instance, we placed the students into small groups (with five to seven members) with a shared journal on Blackboard; each week they used that space to post and discuss their solutions to the weekly homework problem sets, so that they would have the opportunity to see that different solution strategies are often viable. These small groups were chosen with diversity of student backgrounds and academic levels in mind, allowing the sharing of diverse viewpoints in deep discussion that is a hallmark of humanities education in a scalable environment (Hicks has used them effectively in a much larger lecture course).

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## Involvement and Collaboration:

The course is structured to bring together students from across the humanities, the arts, and STEM. While the instructors are both from the humanities, Roby is the liaison for the Classics department's pilot program for the Active Learning Initiative, and the course's pilot offering was included in the [Active Learning Initiative webpage](#). Given the prevalence of STEM departments in the ALI, the innovation grant would provide a natural opportunity to collaborate and share ideas and learning objects with those departments, as well as parallel departments in other institutions. The Classics department's pilot program for the ALI is already strongly focused on developing digital teaching materials that will persist over many cycles of a course offering, so integrating the planned expansions to "The Art of Math" with Roby's existing collaboration with our ALI postdoc will benefit both projects. We have also been in conversation with Peter Lepage, following his visit to our class, about developing a long-term study to measure the effectiveness of

our methods and implement them more widely across the college.

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## Feasibility and Risk:

The course was first offered in Spring 2017. According to self-reporting on the course evaluations (100% response rate), the course attracted students from four colleges – A&S (12), AAP (2), Engineering (2), CALS (1) [two students did not list a college] – and 11 different majors (ranging from Electrical and Computer Engineering to Archaeology, History to Mechanical Engineering). The numerical evaluations were consistently above 4.3, with most above 4.5.

Student comments emphasized a desire to see the course offered again: “This course should definitely be offered again at some point in the future”; “As a math major, I think everyone pursuing math/physics/etc. should be required to take this course”; “The material is very interesting and I hope that the class continues”. Many students commented on the active learning approach that the course provided: “I felt that the approach to the material presented was entirely unique, and so captivating”; “Really enjoyed this class and especially appreciated that it was a space to explore concepts and new areas without fear of judgement. I learned a lot about the subjects discussed as well as thinking about different perspectives”; “Professors Roby and Hicks had great command of the material and created a learning environment that allowed for critical thinking, individual thought, and detailed analysis of the material. They truly demonstrated to me how influential math is in everything in an interesting and stimulating way.”

The problem set discussion journals also reveal that we were successful (in many instances) in encouraging students to reflect on the process of mathematical reasoning:

“When I started this problem set, I had the difficult task of explaining the concepts to myself as much as to a new reader. I wasn't totally convinced I understood the constructions. That being said, I think I have a firmer grasp of the concepts now after working through Proposition 2.

I started by making a list of some general processes that Archimedes uses in his proofs and then applied those concepts to an example. I decided to work with physical paper cuttings as opposed to drawing a diagram for this assignment. The paper cuts helped me really feel the moves that Archimedes made that I couldn't see in a 2-dimensional diagram. I had a hard time following the text of the Method, but I found a wonderful online resource that worked through Proposition 2. (<http://math.furman.edu/~jpoole/archimedesmethod/>)”

And some of the journaling groups developed a rich dialogue. Here is one randomly selected example of the sort of discussion that doesn't often happen in the context of a traditional math class.

**Original post:** I had trouble with question 4, where we had to prove that the angle bisectors of a triangle meet at the triangle's incenter. I tried multiple strategies, but with each one I ended up getting stuck. I also attached the in-class activity from last week, which I think we were supposed to post.

**Reply:** Amanda, your constructions for the fourth question are very clear and well organized. I think it was very clever of you to construct triangles and quadrilaterals. I also tried your first attempt based on the hint and it didn't get me anywhere either.

**Reply:** Yeah, I struggled with how to prove that the angle bisectors were (first of all) actually angle bisectors and (second of all) that the shapes I made within the triangle proved anything about the properties of the angle bisectors. For instance, I knew that the incenter, because it was the center of the incircle, had to be equidistant from any point on the incircle's circumference. But what can this tell me about the angle bisectors?

We look forward to enhancing the tools that we know work, to thinking carefully about modifications in light of student feedback, and to developing new engagement and assessment techniques in dialogue with the CTI.



## Budget:

*(A budget justification form has been provided, please use that to outline line items for the budget. If you have any other comments or narrative around the budget, please do so below.)*

The budget allows us to hire a graduate student and/or ALI postdoctoral assistant with solid computer (but not programming) and mathematical skills (at approx. \$35/hr) to help us expand the current Geogebra notebooks (and other online teaching aids) and integrate them into a newly developed Canvas site for the class. The class materials implemented through this position include computer-based activities designed to give the students the concepts and tools necessary to complete the classwork as well as a facility with Geogebra sufficient to enable its use in in-class Active Learning activities. In particular, we want to build on the Geogebra notebooks we developed last year and build new interactive notebooks as springboards for students to design their own teaching materials. The new Canvas class site will include explanatory videos (produced by media services for approx. \$75/hr) from the instructors walking the students through the demonstrations and proofs, giving students materials that they can review multiple times, on their own and at their own pace, allowing us to spend even more classroom time on hands-on activities. The videos will be designed to ensure maximum flexibility for future iterations of the course, so that they can be used even if the syllabus changes over the years. (CTI offers smaller yearly grants, which could be used if we need to revise or record new videos in the future.) The hands-on in-class activities will be enhanced through the purchase of additional, quality teaching tools (e.g., we have a set of 15 low-cost monochords that sufficed as proof of concept – the students responded very well – but lacked the precision necessary for advanced work). We are keen to explore other hands-on mathematical activities by visiting the National Museum of Mathematics in NYC (the only such museum in North America) and discussing the course with Alexander Jones, Professor of the History of the Exact Sciences in Antiquity at the Institute for the Study of the Ancient World.